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Mathematical Modelling for Environmental Stochasticity on Oxygen-Plankton System

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Abstract – In this talk, we investigate a model of the oxygen-phytoplankton-zooplankton dynamics to understand the underlying properties of the effect of environmental stochasticity both on oxygen production rate and phytoplankton growth rate. We address this issue theoretically by means of a coupled oxygen-plankton dynamics where some parameters are affected from environmental stochasticity. Obtained results show that small noise on oxygen production rate results in less likely depletion of oxygen in a water body and it appears that the probability of oxygen depletion decreases with increasing value of temperature.

Keywords – *Oxygen-plankton system; stochasticity; mathematical modelling; spatial distribution; dynamical system.*

I. Introduction

Variability of ecological process is an attracting phenomenon by field ecologist [2]. It is known that random environmental uctuations can be applied a deterministic model by considering its corresponding stochastic model related to presence of external factors often with arbitrariness [1], [3], [4], [9].

The model that we investigating in previous papers focus on the oxygen production and depletion issue by ignoring natural environmental uctuations [10], [11]. But perfect regularity of a system in theory cannot be explained properly in reality. In order to provide reliable results in terms of uncertainty of the natural phenomenon, the system Dynamics should include variability where in nature environmental factors contain some uctuations and generally called as noise [11]. Systems' dynamical properties can be differ under the effect of noise [5], [7], [8]. Systems' initial conditions sensitivity is increased with noise when its corresponding deterministic system is not show same tendency [4]. In another study, noise in a spatially explicit system can prevent the onset of chaos due to system's sensitivity to the initial conditions [10]. Therefore, here, environmental stochasticity takes our attention to understand its effect on the oxygen-plankton model system.

The aim of this paper is to reveal the role of stochasticity on the dynamics of deterministic oxygen-plankton system [12], [13]. To do this we consider external randomness by modelling question whether environmental fluctuations can be a way to get rid of oxygen depletion.

II. Model System

To understand the systems' behavior under the e_ect of environmental uctuations, it is possible to use below deterministic system (1-3) where the system gain probabilistic properties when some parameters are affected by noise.

$$\frac{dc}{dt} = \frac{Au}{c+1} - c - \frac{uc}{c+c_2} - \frac{\vartheta cv}{c+c_3}, \quad (1)$$

$$\frac{du}{dt} = \left(\frac{Bc}{c+c_1} - \gamma u\right)u - \frac{uv}{u+h} - \sigma u, \quad (2)$$

$$\frac{dv}{dt} = \frac{\beta uv}{u+h}\frac{c^2}{c^2+c_4^2} - \mu v, \quad (3)$$

Due to their biological meaning, all parameters are nonnegative where c is the concentration of oxygen at time t, u and v are the densities of phytoplankton and zooplankton, respectively. Fort he systems' other parameters and the dimensionless version of the paramaters see [12], [13].

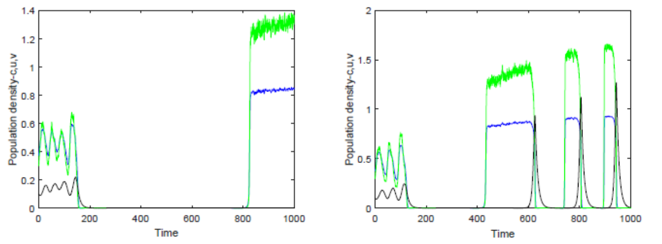
Equation (1-3) are solved for the initial distributions of the system components assumed as fixed at their steady states.

III. Numerical Simulations

Here, we address this problem by interplaying between deterministic oxygen-plankton system (1-3) and noise by taking into account some system parameters are affected by noise, thereby these parameters are not constant any more and depend on time in randomly.

$$c_1 = (c_{1Q} + wd_t) + c_{1D} rand \quad (4)$$

Here random variables are taken into account with the values of rand. Here, the rate of oxygen production is quantified by parameter A with the net oxygen production rate A_Q before global warming, the rate of global warming w, noise intensities random noise.



(a) (b)

Figure 1: Population densities of plankton and concentration of oxygen versus time for different values of w when noise applied to the phytoplankton growth for A = 2.02, c_{1Q}= 0.75, c_{1D}= 0.3 (a) w = 0.0005, (b) w = 0.001, for given initial conditions c0 = 0.385, u0 = 0.3, v0 = 0.1.

Figure 1 shows oxygen and plankton densities for fixed value of A and different values of w. For Fig. 1a, the system components, over the first stage of the dynamics (up to t=170), go to zero after just a few oscillations and then the system densities converge to the zooplankton-free steady state. After staying in zooplankton-free state for a considerable time (approximately between t=200 and t= 800), system trajectories move to oxygen-phytoplankton state. However, for an increase in w, i.e. Fig. 1b, the distribution of oxygen and phytoplankton forms peaks where zooplankton density forms a narrow peak at the position of highest values of oxygen and phytoplankton. Note that the domain is invaded several peaks with same size for increasing values of w.

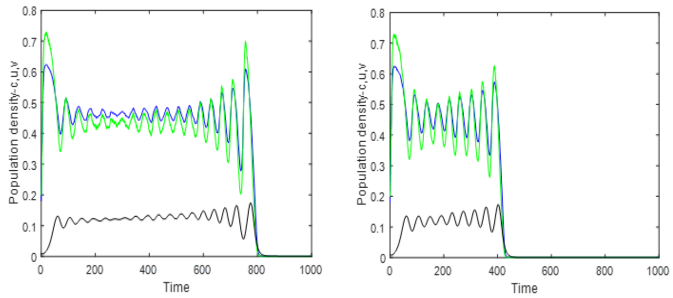


Figure 2: Population densities of plankton and concentration of oxygen versus time for different values of w when noise applied to the phytoplankton growth for A = 2.02, c_{1Q}= 0.75, c_{1D}= 0.1 (a) w = 0.0001, (b) w = 0.002, for given initial conditions c0 = 0.385, u0 = 0.3, v0 = 0.1.

Figure 2 shows oxygen and plankton densities for fixed value of A and different values of w. For Fig.2a, the system components, over the first stage of the dynamics (up to t=800), go to zero after some oscillations and then the system densities converge to the extinction steady state. However, for an increase in w, i.e. Fig. 1b, the exinction happens earlier than Fig.1a. So the important question here is that making some evaluation on the upcoming extinction. Therefore, the system response under some environmental fluctuations detailed in discusson part.

Figure 3 shows oxygen and plankton densities for fixed value of A and same values of w. It means due to random choice of system the system produces diffrent size of oscillations even for exactly same system parameters.

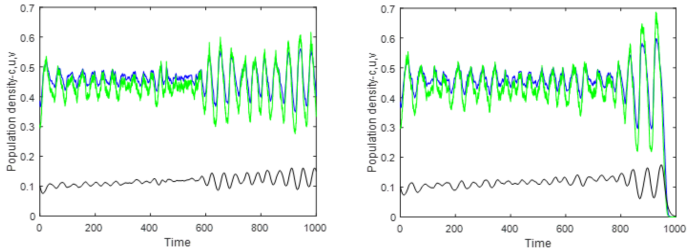


Figure 3: Population densities of plankton and concentration of oxygen versus time for different values of w when noise applied to the phytoplankton growth for A = 2.02, c_{1Q}= 0.85, c_{1D}= 0.3 (a) w = 0.0001, (b) w = 0.0001, for given initial conditions c0 = 0.385, u0 = 0.3, v0 = 0.1.

IV. DISCUSSION

As it is seen here, environmental stochasticity reveals some different properties of oxygen-plankton model. In real world system, all combined action of surrounding water effect on existing plankton system is not possible. So, for the modelling section considering some random parameters make sense to think this model system as in real world.

V. CONCLUSION

In this work, we have studied the oxygen-plankton dynamics using mathematical model taking into account noise where one of the system parameter is affected from noise, i.e. noise applied to the noise applied to the phytoplankton growth. Our main interest is to understand the underlying structure of a nonspatial stochastic inputs effect on oxygen-plankton system whether it affect the distribution and concentartion of plankton species an oxygen depletion, respectively.

We consider a conceptual nonspatial model system corresponds to well-mixed system in real world terms. The system dynamics have been focussed by extensive numerical simulations. We first work on 'average extinction time' for both zooplankton-free state and coexistence state. For increasing value of c_{1D} possible extinction time is delayed. Even if this delayed extinction time under the effect of increasing c_{1D}, the mass extinction is inevitable. We then considered the increasing c_{1D} effect on the average oxygen concentration, i.e. the difference between its maximum and minimum value (max - min). We showed that large increase in c_{1D} results in distinct peaks on population densities.

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