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Mathematical Model on the Temporal and Spatial Structure for Predator-Prey System

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Abstract – In this talk, we work on a model of the prey-predator system to see the effect of system components on species distribution for both temporal and spatial structure. In some cases, changing environmental conditions results in extinction or leads the survival of species. So in this work, the prey-predator system is studied numerically to detail the species sustainability with the assist of gradient addition on systems` parameter. The model consists of two coupled diffusion equations. Extensive numerical simulations on ODE (Ordinary differential equation) and PDE (Partial Differential Equations) are investigated to understand the underlying structure of system response for changing surrounding environmental conditions effect with the assist of system parameters. Obtained results show that the sustainability of species depends on the choice of parameters

Keywords – *Prey-predator system; temporal distribution; spatial distribution; mathematical modelling; simulation.*

I. Introduction

Many theoretical models have been proposed to study the spatial patterning of population dynamics and interactions [7], [8], [14]. Therefore, a considerable progress has been made in understanding the spatial distribution of many species, but an important issue about gradient effect on systems' spatial distributions have not been properly investigated yet. Continuous distribution of species through environmental gradient is detailed with the development of the gradient analysis [1], [15]. There is few literature concerned with gradient effect on spatial patterning. Gleason claim that spatially heterogeneous patterning can be explained by individuals' spatial gradient response [4], [5], [6]. On the other hand, spatial gradient effect on herbivore population dynamics is focussed in [13]. In particular, Pascual considered spatial gradient by taking into account it both for prey and predator diffusion terms in [10]. Geographical patterns and ecological processes of speciation along environmental gradients is focussed by evolutionary branching in spatially
structured populations by [2].

In this work, we consider a predator-prey model system where predator mortality rate decreases with the assist of spatial gradient. Numerical results show that the systems' species extinction is followed by regular pattern formation in spatial system; see similar tendency to detect "path to extinction" [16]. We construct that the system has a map in a parameter plane (x_1,w), i.e. beachhead and slope of the gradient, respectively, with different domains corresponding to species extinction and persistence. On the other hand, for the corresponding temporal distributions for both prey and predator show oscillations with different size. In particular, for different choice of initial distributions for both prey and predator, the system spatial distributions do not change and keep its extinction point where the whole system goes extinct. It means, travelling pulses die out by-one-by when they reach systems' critical point. In this case extinction is inevitable.

II. Mathematical Model

We begin the following predator-prey interaction model:

$$\frac{du}{dt} = \gamma u(u-\beta)(1-u) - \frac{uv}{1+\alpha u}, \qquad (1)$$
$$\frac{dv}{dt} = \frac{uv}{1+\alpha u} - \delta v, \qquad (2)$$

(cf. [12]). Here u and v are the densities of prey and predator, respectively, at time t and position x. D is diffusivity; for more details on system parametrizations and its dimensionless version see [12]. Here the positive part of prey is for growth and the negative part is for losses. Same idea is valid for predator as well.

III. Numerical Simulations

Equations (1-2) are solved numerically on the domain $0 < x < L$ by finite difference method using the zero flux boundary conditions. The steps of the numerical mesh are chosen as $\Delta x = 0.5$ and $\Delta t = 0.01$; it was checked that these values are small enough not to results in any numerical artifacts. Since we are mainly interested in spatial gradient effect on system dynamics, we keep the system parameters as in [12]. System parameters for both temporal and spatial structure is obtained for following parameters α= 0.5, β= 0.28, γ= 7, Δu = 5, Δv =3, u_0 = 1, v_0 = 1 and the domain for spatial structure L = 400. First we consider initial conditions as in the following form:

$u(x, 0) = u_0$; for $-\Delta u < x < \Delta u$; otherwise $u(x; 0) = 0$; (3)
$v(x, 0) = v_0$; for $-\Delta v < x < \Delta v$; otherwise $v(x; 0) = 0$ (4)

where u_0, v_0 are the initial population densities and Δu, Δv correspond the initially invaded domain radius. Initial conditions are given by Eqs. (3-4) correspond to biological control problem [3], [9], [11]. To check the system (1-2) spatial distribution dependence to the initial conditions, different type of initial conditions are used. In particular, in the case of the following constant gradient type, i.e.,

$$u(x,0) = \tilde{u}\,; \qquad v(x,0) = \tilde{v} + \varepsilon\,(x - x_0) \tag{5}$$

where $\tilde{u}$, $\tilde{v}$are the steady states of the system dynamics, given by Eq. 5, with ε = 0.001 and x_0 = 200, i.e. auxiliary parameters.

$$\delta = \delta_L - w(x - x_1) \tag{6}$$

Here x_1 is beachhead, δ_L is the predator mortality rate before changes, and parameter w quantifies the slope of the spatial gradient.

In order to understand the underlying structure of gradient effect on species extinction, we chose the initial mortality rate of predator δ_L, where δ corresponds to species extinction without gradient case; see parameter plane map (Fig. 13) in [12].

Temporal distribution of the system is given in Fig.1. For the increasing values of δ the system evolve as an extinction to limit cycle state. As it is seen in the top left Fig. 1; the system goes extinct and then an increase in δ results in limit cycle and this limit cycle decrease in size with and increase in this parameter value.

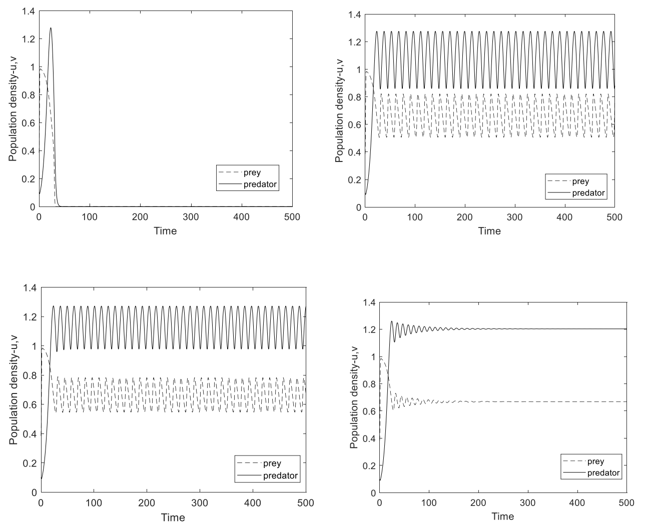

Figure 1: Temporal distribution of prey (dashed)-predator (solid) density for t=500 (time). For given parameters α= 0.5, β= 0.28, γ= 7. For different δ *values as* $\delta = 0.494$; $\delta = 0.495$; $\delta = 0.496$; $\delta = 0.501$ from left to right top to bottom.

The spatial distribution of prey-predator system for corresponding temporal distribution is given as in Fig.2. Other corresponding values with temporal structure is not given here due to systems` similar behaviour. It is observed that the smooth pattern is followed by distinct peak. These peaks reach their max value around the middle of the domain and then when they converge to the end of the domain their height decrease. Therefore, checking the height of these peaks can give an idea on underlying structure of these system.

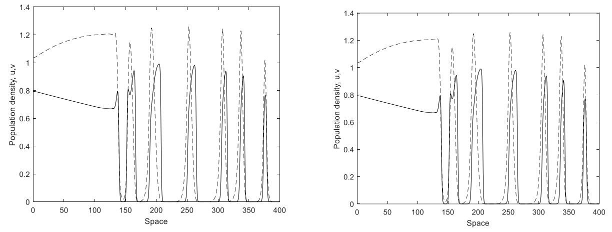

Figure 2: Spatial distribution of prey-predator system for given domain L=400 and for given time moment t=500. System parameters are exactly same with temporal structure as in given in tex and $\delta = 0.494$; $\delta = 0.501$.

IV. Results

In this work it is shown that predator-prey system [12], the spatiotemporal dynamics can become persistent under the infuence of the spatial gradient in predator mortality rate. A predator-prey system with the gradient addition can exhibit such patterns of travelling pulses.

V. DISCUSSION

This kind of structural behaviour should be checked that to obtain the similar tendency in other system. Also, the structural behaviour of the system in two-dimension should be detailed to compare its one-dimensional case.

VI. CONCLUSION

By means of numerical simulations, we revealed the structure of system spatial distributions for linearly decreasing values of δ in space. This spatial structure gives important information when the predator mortality rate is closer to the extinction point. Here we observe the same spatial distributions, i.e. patchy to regular, with oxygen-plankton system (see [16]), when system species go to extinction.

The obtained results shown that species persistence can be interpreted by the slop (steepness) of the spatial gradient and the beachhead distance. If the gradient is increased, the system persistence is decreased. In this case, if the beachhead is taken suffciently large, the population goes extinct. It is also observed that, there is a recovery on predator-prey system under the effect of spatial gradient. Same results were obtained for oxygen-plankton model system under the effect of climate change [16].

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