

Modelling Flood Routing Using Hybrid Heuristic Algorithm

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Abstract –In the analysis of the flood routing problem, it is necessary to determine the change of the flood wave moving through the channel according to time and location, and in this context, nonlinear 3-parameter Muskingum model (NL-MUSK) is widely used. The parameters of the NL-MUSK model should be estimated precisely to hold the appropriate solutions. In general, this important process is solved by the use of heuristic methods, one of which is the Particle Swarm Optimization (PSO). This study aims to improve searching the local solution capability among the possible global solutions revealed by the particle swarm optimization (PSO) in a NL-MUSK flood routing problem. To this end, the hybrid use of the PSO with the Levenberg-Marquardt (LM) algorithm, based on Jacobian matrix, was used. The developed algorithm (PSO-LM) was applied to the flood data observed in 1960 on the River Wye, U.K. The hybrid PSO-LM algorithm was operated under a wide solution space and the same global solution was obtained from each random experiment. The PSO-LM, which stands out with its stable aspect, has also achieved rapid convergence compared to the PSO, in other words, it has accelerated the access of the model to the optimum result with less iteration. In this respect, it is concluded that PSO-LM can be adapted effectively to different hydrological modeling studies.

Keywords –Nonlinear Muskingum model, Hybrid heuristic methods, Parameter estimation, Levenberg-Marquardt algorithm

I. INTRODUCTION

In the water resources engineering, flood routing is a required procedure to predict the variation in a flood wave characteristics for natural rivers or channels. Although the employed methods in flood routing display diversity, Muskingum model among them is definitely more practical since it allows estimating the outflow as a function of the inflow without needing any topographical input pertaining to river channel [1].

The Muskingum model, which was developed by McCarthy [2], is based on two equations that demonstrate the continuity equation (Eq.1) and storage equation (Eq.2) as follows:

$$\frac{dS_t}{dt} = \Delta S_t = I_t - Q_t \quad (1)$$

$$S_t = a[xI_t + (1-x)Q_t] \quad (2)$$

In the above equations, S_t , I_t and Q_t are storage, inflow and outflow magnitude, respectively at time t . a is storage-time constant and x is a weighting factor [1]. In Eq.(1), ΔS_t is the assumed as time rate of change of storage volume.

Since the first developed Muskingum method assumes that the storage volume has a linear relation with the weighted value of inflow and outflow, and that cannot be feasible for all cases, Gill [3] suggested a three-parameter nonlinear Muskingum model (NL-MUSK), in which modified storage function was defined as Eq.(3).

$$S_t = a[xI_t + (1-x)Q_t]^m \quad (3)$$

This version contains an additional power parameter m to promote the statistical relation between accumulated storage and weighted flow.

In literature, there are numerous studies considering the parameter estimation of the NL-MUSK, in which the unknown parameters are a , x and m , respectively. Since the classical techniques (e.g., derivative Newton methods, simplex algorithms) can fall into local minima traps, the recent works have been focused on the population based heuristic algorithms to get global minimum. The literature summary about this issue was comprehensively handled by Karahan et al. [4] and Moghaddam et al. [5]. According to investigations, it was detected that usages of heuristic methods in NL-MUSK modeling did not guarantee the same solution under different runs. Therefore, some researchers have tried out the hybrid optimization techniques that combine the global searching abilities of heuristic algorithms with local fine-tuning properties of the non-heuristics such as Quasi-Newton [4] and Nelder-Mead simplex [6].

Even if all these efforts increase the possibility of getting global solution, these combinations made the algorithmic structure intensive in terms of the multiple control variables. Therein, in this study, we only focus on the particle swarm optimization (PSO) algorithm, which is one of the most practical methods among the other heuristics. Besides its inherent capability, the Levenberg-Marquardt (LM) algorithm is also embedded into it in order to improve the local searching abilities and enforce stabilities during generations. The content of the proposed hybrid approach PSO-LM, and the details about NL-MUSK and application are given below.

II. METHOD

Routing in NL-MUSK

In the NL-MUSK, the initial storage (S_0) is primarily calculated by using Eq.(3). At $t=0$, the outflow can be assumed to be equal to the inflow. Then, the time rate of change of storage volume ΔS_t is computed. Thus, the amount of storage in the next step can be determined. Finally, the next modeled outflow (Q_{t+1}) is calculated by using Eq.(3). In the parameter optimization step, the objective (fitness) function to be minimized within the employed algorithms is the sum of the squared errors between computed and measured outflows (SSE). In case infeasible parameters give negative outflows and storages, a penalty function is also used. The detailed computing steps of NL-MUSK and penalty function usage are given by Karahan et al. [4].

Proposed Hybrid PSO-LM Algorithm

The PSO, which is typically based upon the social behavior of swarm of birds, is a population-based heuristic algorithm that was proposed by Kennedy and Eberhart [7]. The PSO procedure starts with a random distribution of swarm of birds into the food area. A matrix composed of N_p particles and d parameters can be written in matrix form as follows:

$$\vec{X} = \begin{bmatrix} x_{1,1} & \dots & x_{1,d} \\ \vdots & \ddots & \vdots \\ x_{N_p,1} & \dots & x_{N_p,d} \end{bmatrix} \quad j=1,2,\dots,d ; i=1,2,\dots,N_p \quad (4)$$

To find out the locations of the food sources, particles interact with two different terms. The first of these terms, p_{best} , represents the best location information that any particle has ever reached, while the second term, g_{best} , denotes the best position obtained from all the particles in the population. In the algorithm, there is also a vector v which gives the iterative velocity variation of the particles (Eq.5). Then, with Eq.(6), the particles are allowed to move to their new position.

$$v_{i,j}(t+1) = W * v_{i,j}(t) + r_1 c_1 * (p_{best_{i,j}}(t) - x_{i,j}(t)) + r_2 c_2 * (g_{best_j}(t) - x_{i,j}(t)) \quad (5)$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \quad (6)$$

Here, W is the inertia weight which linearly changes from 0.9 to 0.4. On the other hand, c_1 and c_2 are positive acceleration coefficients controlling p_{best} and g_{best} components, while r_1 and r_2 refer to random numbers that are uniformly distributed between 0 and 1. The above operations are repeated between $t=0$ and the maximum number of iterations ($iter_{max}$).

In the study, the g_{best} component of the PSO was exposed to the LM algorithm to improve the local search behaviour of the process. LM algorithm is operated with the Jacobian matrix (J) which is composed of first order partial derivatives according to the related parameters. After a finite difference approach is evaluated for calculation of J matrix, in each iteration step, the parameter updating process is efficiently operated through this hybrid approach termed as PSO-LM. This modification directly increases the convergence performance. So, the advantageous aspects of the two

methods have been revealed, while the shortcomings of them have been disabled.

III. RESULTS

To examine the credibility of the suggested hybrid PSO-LM technique and to compare it with the PSO, an example of the 1960 flood event in the River Wye at U.K. was tested in this study. Because the 69.75 km stretch of the River Wye from Erwood to Belmont has no tributaries and minimal lateral flow, it is an illustrative case to test flood routing applications [4]. The related data can be extracted from Karahan et al. [4]. The inflow and outflow hydrographs with regard to occurred event are given in Fig.1.

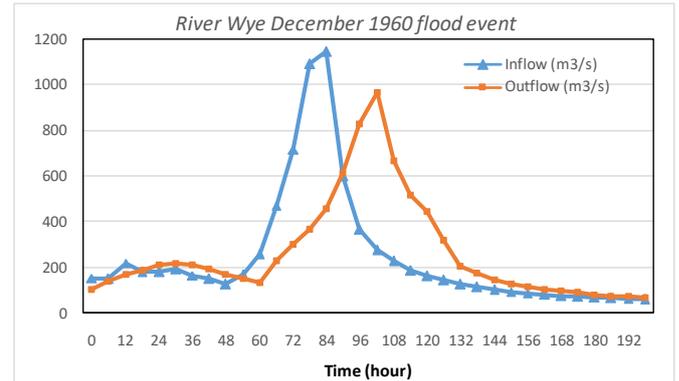


Fig. 1 River Wye 1960 flood hydrographs used in the study

The employed algorithms were coded in the MATLAB environment. The ranges of the parameters were selected as $x=0.01-0.5$, $a=0.01-5.0$, and $m=0.01-5.0$. Although such algorithms were run with huge number of iterations in the literature, a feasible number of generation were used in this study similar to Niazkar and Afzali[8], and $iter_{max}$ was set to 500. In addition, the population size N_p was taken as 20.

Due to stochastic features of algorithms, 100 independent runs were operated to detect the performances. After operating hybrid approach depending on the strategy stated in the previous section, the variations concerning fitness values obtained for each run were plotted during 500 iterations (Fig 2).As seen in Fig. 2, there are some runs in the PSO that do not produce stable results. Otherwise, it can be clearly seen that the PSO-LM simulations rapidly converge to the global result during iterations. To support this graphical detection, the fitness performances of the algorithms at the last iteration were examined in detail. The fitness values stored for each run were extracted and then some descriptive statistics (mean, standard deviation) were obtained.

The SSE statistics given in Table 1 indicate that the PSO can get trapped in a local minimum. In contrast to the PSO, PSO-LM has reached almost the same SSE fitness value in all of the simulations because the standard deviation of fitness values was calculated as $1.93E-08$. Thus, after utilizing the global search capability and practical structure of the PSO, each solution vector obtained from its iterative mechanism was subjected to the LM process alternately. Moreover, an efficient search was performed without being affected by the local minima. When it is compared with the other studies in point of iteration settings, e.g. Yuan et al. [9], the proposed PSO-LM is absolutely applicable. However, it should be noted that the selection of the iteration number is not only a difficult task but also problem specific.

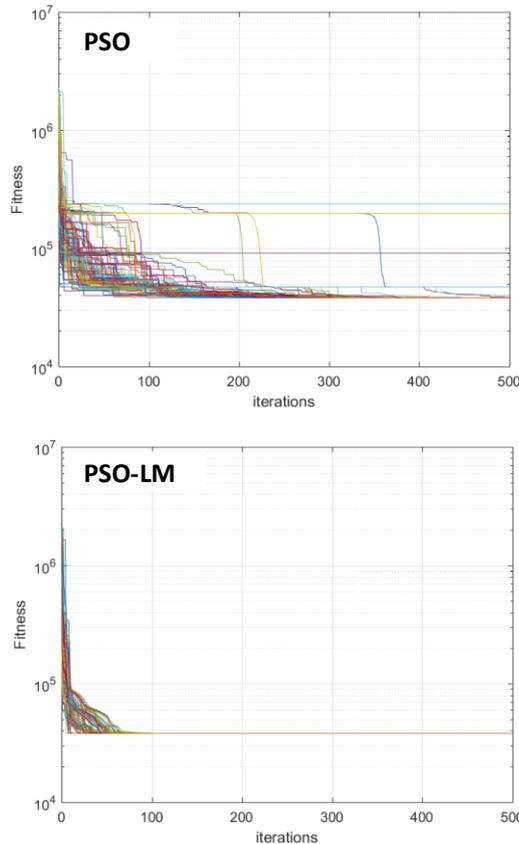


Fig. 2 The change in the fitness values through iterations under PSO and PSO-LM approaches

In this study, the success of the NL-MUSK model was also investigated by the additional measures. When referencing Niazkar and Afzali [8], DPO criterion, which is expressed as the absolute value of the deviation in peak of observed and routed outflows, can be used for magnitude of peak consideration.

Table 1. SSE statistics of the algorithms for $N_p=20$ and $iter_{max}=500$

| Descriptive Statistics | PSO | PSO-LM |
|------------------------|----------|----------|
| Mean | 59822.43 | 37944.14 |
| Standard deviation | 52878.38 | 1.93E-08 |

The coefficient E , which is a modification of the determination coefficient, was also evaluated. The ultimate parameter estimations obtained from PSO-LM and the measures were given in Table 2.

Table 2. The calibrated parameters of PSO-LM and the related statistics

| Calibrated Parameters | | | Statistical Measures | | |
|-----------------------|---------|---------|----------------------|--------|-----------------|
| x | a | m | SSE (m^6/s^2) | E | DPO (m^3/s) |
| 0.40924 | 0.07924 | 1.58148 | 37944.1446 | 0.9771 | 97.808 |

On account of the fact that the performance limits $E>0.75$ and $RSR<0.5$ have been met in accordance with Moriasi et al. [10], these computed measures point out a high-performance model class. As shown in Fig 3, it can be turned out again that both observed and predicted outflows are soundly matched. Additionally, the DPO criteria, which are evaluated in the peak value estimation of the outflow hydrograph, also appear to be acceptable.

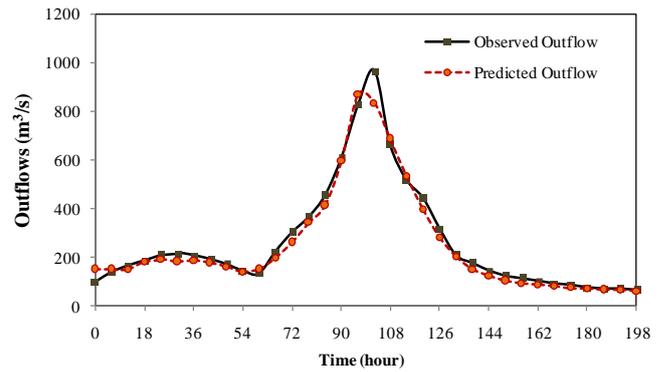


Fig.3. Comparison of the observed and predicted outflow hydrographs

IV. CONCLUSION

In the literature about parameter calibration of the NL-MUSK, researchers may discover a number of applications varying from the derivative-based algorithms to heuristics. Although heuristics are evaluated as more popular in terms of their global search capabilities, in many cases, with the heuristic algorithms that require more control parameters than the number of parameters in the NL-MUSK, the performed applications have begun to move away from practicality. In this regard, this study introduces a practical hybrid approach that is constituted with the PSO and the LM algorithms together. To examine the reliability of this suggested technique, River Wye data was used. The comparisons between the PSO and PSO-LM revealed that the hybrid algorithm satisfied both the stability in each random simulation and the fast convergence for the generations. According to the statistical measures derived from the implementation, it was concluded that both observed and predicted outflows were properly matched. Additionally, it is thought that the PSO-LM algorithm can be applied to various modelling studies in water resources engineering.

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