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Cauchy Multiplication by Nörlund Summability Generalized Nörlund Summability

Şaban Yılmaz*

[1]*Department of Computer Technologies / Turhal Meslek Yüksekokulu, Gaziosmanpaşa University, Tokat, Turkey*
**Corresponding author: saban.yilmaz@gop.edu.tr*
+Speaker: saban.yilmaz@gop.edu.tr
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Abstract **–** The definition set is called arrays of functions consisting of natural numbers. The sequence of the arrays is called a convergent sequence to give a real number result, otherwise it is called a divergent array. In 1897, A. Tauber aimed to classify the conditions under which the sequences are convergent. Tauber emphasized the conditions that provided convergence to his work. Among the investigations, a remarkable feature is the convergence of divergent knees. In 1905, E. Cesàro, which was the first to draw attention among these studies, revealed the theory of intergroup transformation. The basic principle of this transformation was iy to convert the convergent sequence into a convergent sequence and to maintain its limit Bu. This clutch is called regularity. Cesàro summability provides regularity and diverging some divergent sequences into convergent sequences. In 1910, L. L. Silverman expressed and proved the theorem in 1913, which transformed the convergent sequence into a convergent sequence and revealed its conditions. In 1911, M. Riesz defined a new transformation on any series with positive terms. N. E. Nörlund described the Nörlund summability by making a similar transformation in 1920. In the following years, generalization of Nörlund summability has emerged. In this study, Cauchy products of Nörlund summability generalized with Nörlund summability were studied. It has been observed that the Cauchy product of the Nörlund summability generalized with the Nörlund average transforms to the average of Nörlund.

Keywords – Riesz Summability, Cesàro *Summability, Generalied Nörlund Summability.*

I. Introduction

Definition 1. All non-zero and non-negative numbers are given a (p_n) sequence.

P_n sequence of partial sums

$$P_n = p_0 + p_1 + p_2 + \cdots + p_n \ , p_0 > 0 \ , (n \in \mathbb{N})$$

and $\sum_{n=0}^{\infty} u_n$ series, including (s_n).

The (τ_n) sequence given by $\tau_n = \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}s_k$ is the transformation obtained by the Nörlund summability of the sequence s, which is called Nörlund summability.

Accordingly, if an ℓ value is provided that provides the equation of

$$\lim_{n\to\infty} \tau_n = \lim_{n\to\infty} \frac{1}{P_n}\sum_{k=0}^{n} p_k s_k = \ell$$

then the o s sequence is ℓ that can be summed to N^p li.

Definition 2. Let (p_n), (α_n) be a sequence of all non-zero and non-negative numbers. For each $n \in \mathbb{N}$ and given partial series of a given series

$$P_n^{\alpha} = \sum_{k=0}^{n} p_{n-k}\alpha_k \ , \quad P_{-n}^{\alpha} = 0$$

$\sum_{n=0}^{\infty} u_n$ is called the conversion sequence obtained by the generalized Nörlund summable of the s sequence to the (s_n^*) sequence given by

$$s_n^* = \frac{1}{P_n^{\alpha}}\sum_{k=0}^{n} p_{n-k}\alpha_k s_k \ , \ n \in \mathbb{N} \tag{1}$$

(Nurcombe, 1989). This transformation corresponds to matrix $N^{p\alpha} = (a_{nk}^{p\alpha})$; It is defined as

$$a_{nk}^{p\alpha} = \begin{cases} \dfrac{p_{n-k}\alpha_k}{P_n^{\alpha}}, & k \le n \\ 0 & , k > n \end{cases}$$

for $k, n \in \mathbb{N}$.

Definition 3. Let a matrix be given. The matrix A is called regular if the matrix A converts each convergent sequence into a convergent sequence and at the same time maintains its limit. (Petersen, 1966)

Lemma 1. (Silverman-Teoplitz theorem) matrix to be regular,

i) $\|A\| = \sup_{n\in N} \sum_{k=0}^{\infty} |a_{nk}| < \infty$

ii) For each k $\lim_{n\to\infty} a_{nk} = 0$

iii) $\lim_{n\to\infty} \sum_{k=0}^{\infty} a_{nk} = 1$

the conditions are necessary and sufficient (Mears, 1937).

Lemma 2. There are $\left(\sum_{k=0}^{\infty} a_k\right)\left(\sum_{k=0}^{\infty} b_k\right) = \sum_{k=0}^{\infty}\sum_{i=0}^{k} a_{k-i} b_i$ equality, including (a_n) and (b_n) two sequences. This equation is called the Cauchy product. (Nesin, 2011)

II. Materials and Method

Theorem 1. The generalized Nörlund summability of the (s_n) sequence to the (s_n^*) sequence can be aggregated to the Z value generalized Neurlund and the (τ_n) sequence can be collected to the t^* of the (t_n) sequence to the Nörlund. In this case, the Cauhcy multiplications of the (s_n^*) and (τ_n) sequences are Nörlund collected to the $s^* t^*$ value.

Proof: $s_n^* = \frac{1}{P_n^{\alpha}} \sum_{k=0}^{n} p_{n-k}\alpha_k s_k$, $\tau_n = \frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} t_k$,

using the Cauchy product,

$$s_n^* t_n^* = \left(\frac{1}{P_n^{\alpha}} \sum_{k=0}^{n} p_{n-k}\alpha_k s_k\right)\left(\frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} t_k\right)$$

$$= \frac{1}{P_n^{\alpha} P_n} \sum_{k=0}^{n}\sum_{i=0}^{k} p_{k-i}\alpha_i s_i q_i t_{k-i}$$

$$= \left(\frac{1}{P_n^{\alpha}} \sum_{k=0}^{n} p_{n-k}\alpha_k\right)\left(\frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} s_{n-k} t_k\right)$$

$$= \left(\frac{1}{P_n^{\alpha}} P_n^{\alpha}\right)\left(\frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} s_{n-k} t_k\right)$$

$$= \frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} s_{n-k} t_k .$$

Here we get the $s_n^* t_n^* = \sum_{k=0}^{n} g_{nk} s_{n-k} t_k$ equation for $k, n \in \mathbb{N}$, when the

$$g_{nk} = \begin{cases} \dfrac{p_{n-k}}{P_k} & , k \le n \\ 0 & , k > n \end{cases}$$

and $G = (g_{nk})$ matrix are defined. This G-picking method is also regular when the neuron aggregability is regular. That's why it's

$\lim_{n\to\infty} g_{nk} = 0$.

Since a regular matrix will convert the convergent sequence into a convergent sequence and maintain its limit,

$$\lim_{n\to\infty} \sum_{k=0}^{n} g_{nk} s_{n-k} t_k = s^* t^*$$

is obtained.

III. Results

In this study, it was obtained that Nörlund aggregability and generalized Nörlund aggregability were reduced to Nörlund summability when Cauchy products were made.

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