

## Solving p-Hub Median Problem via Artificial Bee Colony Algorithm Considering Different Search Strategies

Betül YILDIRIM<sup>1,2\*</sup>, Latife GÖRKEMLİ<sup>2</sup>

<sup>1</sup>Industrial Engineering Department, Nuh Naci Yazgan University, Kayseri

<sup>2</sup>Industrial Engineering Department, Erciyes University, Kayseri

\*(byildirim@nny.edu.tr)

**Abstract** –The hub location problem deals with locating hub facilities and allocating non-hub nodes to hub. Hub location problem is classified as p-hub median problem, p-hub center problem, hub covering problem and hub location problem with fixed cost in the literature. In this paper, multiple allocation p-hub median problem type is discussed. Artificial bee colony algorithm is used to solve multiple allocation p-hub median problem. The artificial bee colony algorithm which is originally developed for solving continuous optimization problems is adapted to handle the discrete structure of the hub location problem. Different search strategies are considered in order to obtain more efficient solutions. With individually coded approaches, the performances of the algorithm are tested and its effectiveness are demonstrated.

**Keywords** – Hub Location Problems, p-Hub Median Problems, Artificial Bee Colony Algorithm, Optimization, Multiple Allocation

### I. INTRODUCTION

Hubs are centers between origin and destination point. These points are called nodes in a given network. The hub location problem deals with locating hub facilities and allocating non-hub nodes to hub. The hubs offer the possibility of transferring units from resource points to target points using less direct connection and cost. In order to achieve this, the hubs must be correctly positioned and the nodes assigned to these hubs appropriately. Air transportation problems, cargo delivery problems, telecommunication problems are examples of the hub location problems.

Hub location problems are grouped as p-hub median problem, p-hub center problem, hub covering problem and hub location problem with fixed cost in the literature. Three assumptions are made in the literature about the hub location problems[1]:

- Allowing direct connection between the two hubs,
- Utilizing economies of the scale in network flows,
- Not allowing direct connection between non-hub nodes.

The first study on the hub location problems was done by O’Kelly in 1986, involving one and two hubs. The study focused on mathematical model development [2].

In this paper, uncapacitated multiple allocation p-hub median problem (UMApHMP) is discussed. In this problem, N nodes and the flows between all pairs of nodes are defined in the network structure. The number of hubs

to be opened is determined as p. Multiple allocation allows each node to be allocated to several hubs. This problem is known to be NP-hard. In this problem, the goal is to minimize the total transportation cost.

Campbell [3] developed the first linear integer mathematical model for UMapHMP. Then, Campbell [4] developed a mathematical model that takes into account the flow threshold and fixed costs for connection lines in this problem.

The first studies for solving the hub location problem are related with mathematical model development. And up to now various algorithms (ant colony algorithm, genetic algorithm, simulated annealing algorithm) have been developed in order to solve these problems more effectively.

In this study, artificial bee colony algorithm is used to solve multiple allocation p-hub median problem. In recent years, artificial bee colony algorithm has been used frequently in solving optimization problems and gathering successful results. This algorithm which is originally developed for solving continuous optimization problems is adapted to handle the discrete structure of the hub location problem. Different search strategies are considered in order to obtain more efficient solutions.

### II. MATERIALS AND METHOD

In this paper multiple allocation p-hub median problem is discussed. And artificial bee colony algorithm (ABC) is used to solve this problem. The definition of considered problem and the basic steps of ABC are given in below.

### A. Problem Description

In this problem structure, there are N nodes in the network. The aim is to determine the hubs, assign the non-hub nodes to hubs and minimize the total transportation cost.

In this type of problem, transfers between nodes can occur as follows:

- Nonhub-hub
- Hub-hub
- Hub-nonhub
- Nonhub-nonhub (via a hub or hubs)

There are many mathematical models developed in the literature for this type of problem. The mixed integer linear programming model developed by Boland et. al. [5] is as in the following.

Decision variables;  
 $H_j$  : 1, if j is hub; 0 otherwise ,  
 $Y_{kl}^i$  : the amount of flow from node i, collected at hub k, transported via hub l,  
 $Z_{ik}$  : the amount of flow from node i, collected at hub k,  
 $X_{lj}^i$  : the amount of flow from node i to node j, distributed via hub l,  
 $W_{ij}$  : the flow between node i and node j,  
 $C_{ik}$  : the dimension between i and k.

Model:

Min

$$\sum_{i=1}^n [\lambda \sum_{k=1}^n C_{ik} Z_{ik} + \alpha \sum_{k=1}^n \sum_{l=1}^n C_{ik} Y_{kl}^i + \gamma \sum_{l=1}^n \sum_{j=1}^n C_{lj} X_{lj}^i]$$

$$\sum_{i=1}^n H_i = p \quad (1)$$

$$\sum_{k=1}^n Z_{ik} = \sum_{j=1}^n W_{ij} \quad \forall i \quad (2)$$

$$\sum_{l=1}^n X_{lj}^i = W_{ij} \quad \forall i, j \quad (3)$$

$$\sum_{l=1}^n Y_{kl}^i + \sum_{j=1}^n X_{kj}^i - \sum_{l=1}^n Y_{lk}^i - Z_{ik} = 0 \quad \forall i, k \quad (4)$$

$$Z_{ik} \leq \sum_{j=1}^n W_{ij} H_k \quad \forall i, k \quad (5)$$

$$\sum_{i=1}^n X_{ij}^i \leq \sum_{i=1}^n W_{ij} H_l \quad \forall l, j \quad (6)$$

$$X_{lj}^i, Y_{kl}^i, Z_{ik} \geq 0, H_k \in \{0,1\}$$

When the objective function of the model is analyzed, it is seen that total collection, transfer and distribution costs are minimized by using coefficients such as  $\lambda, \alpha, \gamma$ . The first constraint ensures that the total number of hubs to be opened is up to p. Constraints 2-4 represent the conserving equations for the network flow problem for each node i. Constraints 5 and 6, prevent the direct connection between two nodes without hub. The last constraint indicates that flows cannot be negative, and binary nature for decision variables [6].

### B. Artificial Bee Colony Algorithm (ABC)

Karaboğa [7] developed the artificial bee colony algorithm inspired by the intelligent foraging behavior of bees in nature. In this algorithm, the bees in the colony are divided into three groups as employed bees, onlooker bees and scout bees. Half of the colony consists of employed bees, while the other half consists of onlooker bees. The number of food sources is equal to number of employed bees. When the amount of nectar in the sources is over, employed bee become scout bee.

The main structure of the ABC algorithm are given below [8]:

*Initialization Phase*

**REPEAT**

*Employed Bee Phase*

*Onlooker Bee Phase*

*Scout Bee Phase*

*Memorize the best solution*

**UNTIL** (max cycle number)

In ABC algorithm, each cycle consist of three steps [9]:

- Sending employed bee to food source and collecting nectar from the sources,
- Selection of food sources by scout bee based on information from the employed bee,
- Identifying scout bees and sending them to look for new food sources.

In the initial phase, the values of the control parameters (limit, colony size, maximum cycles) are determined. Then, initial solutions are generated randomly with Eq. 1.

$$x_{ij} = x_j^{min} + \text{rand}(0,1)(x_j^{max} - x_j^{min}) \quad (1)$$

In this equation,  $i=1, \dots, SN$  and  $j=1, \dots, D$ ; SN is number of food sources and D is number of parameters.  $x_j^{min}$  is lower bound and  $x_j^{max}$  is upper bound of parameter j [9]. In the employed bee phase, employed bees search for new neighbor sources using Eq. 2.

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) \quad (2)$$

where k is number of food sources,  $\phi$  is a random number within the range [-1, 1]. Then fitness values of the solutions is calculated with Eq. 3.

$$\begin{aligned} fit_i &= \frac{1}{1+fi} & fi &\geq 0 \\ fit_i &= 1 + \text{abs}(fi) & fi &< 0 \end{aligned} \quad (3)$$

After calculating fitness values, employed bees apply greedy selection between old and new food sources. Best solution kept in memory. After this phase, depending on employed bees information, onlooker bees probabilistically choose food sources by Eq. 4 [10].

$$p_i = \frac{fit_i}{\sum_{n=1}^{SN} fit_n} \quad (4)$$

Then onlooker bees select the food sources according to their probability values and create new sources by using Eq. 2. If the solution cannot be improved until the limit value, scout bees search the new food sources.

**C. Artificial Bee Colony Algorithm for *p*-Hub Median Problem**

In the proposed algorithm solution consists of selected nodes as hubs. For example; in a problem with 10 cities, when nodes 4, 7 and 8 are selected as hubs, the solution representation is expressed as {4, 7, 8}. The fitness value of the solution is calculated in the objective function of the given mathematical model in paper.

**Strategy 1:** First, a clustering mechanism which is appropriate for the used data set in the paper is considered. In this mechanism, the search space is divided into *p* clusters. According to this mechanism, the total distance from a city to all cities are calculated for each other firstly. Then the cities are sorted in decreasing order considering distances. The number of  $N/p$  cities are placed into clusters considering the distance order. Hubs are randomly selected from each cluster and the initial set of solutions is created. For example; for the two hubs problem, when searching for neighborhoods in the employed bee phase, the first initial hub remains the same, the second hub is replaced with another randomly selected hub from the same cluster itself. In the onlooker bee phase, the second hub remains the same, the first hub is replaced similarly. In three hubs problem one of the hubs is selected and changed randomly and in four hubs problem two of the hubs are selected and changed randomly in both phases.

**Strategy 2:** This strategy focuses on random selection. *P* number of nodes are randomly selected from network. For each solution, unselected nodes are assigned to a cluster called *nonhub*. Here, the hub switching mechanism is formed by changing the randomly selected hub in the solution with randomly selected node from the *nonhub*. In both phases same mechanism is used and *nonhub* is updated according to the changes.

**Strategy 3:** The third strategy is developed as an alternative to the neighborhood search method of Strategy 1. In the neighborhood search of this strategy all the hubs are changed. Each hub is replaced with another randomly selected hub from the same cluster itself. In both two phases this situation is considered. In Strategy 1 and Strategy 3, clusters are not considered when creating new food sources in the scout bee phase. In these new solutions hubs are selected randomly.

In all strategies, the node assignments to the hubs are done according to the Floyd-Warshall algorithm.

Limit, colony size, maximum cycles are important control parameters that affect the performance of the ABC algorithm. In this study, the appropriate parameters are determined as a result of various analysis. These values are given below:

- Colony size=  $N*2$
- Limit=20
- Maximum cycles=20

For each strategy the problems were run 30 times in MATLAB. The results are shown Table 1 and Table 2.

Table 1. ABC Result1

n p α	optimum	Strategy 1		Strategy 2		Strategy 3	
		mean	score	mean	score	mean	score
20 2 0,2	972,251	972,251	30	972,251	30	972,251	30
20 2 0,4	1.013,358	1.013,358	30	1.013,358	30	1.013,358	30
20 2 0,6	1.046,895	1.046,895	30	1.046,895	30	1.046,895	30
20 2 0,8	1.075,301	1.075,301	30	1.075,301	30	1.075,301	30
20 2 1,0	1.090,628	1.090,628	30	1.090,628	30	1.090,628	30
20 3 0,2	712,090	713,568	10	721,690	21	712,090	30
20 3 0,4	803,810	811,635	12	806,910	25	803,810	30
20 3 0,6	884,636	895,911	25	887,002	24	884,636	30
20 3 0,8	948,415	951,487	22	950,217	17	949,694	28
20 3 1,0	975,532	976,435	28	979,818	19	975,532	30
20 4 0,2	568,505	600,143	4	598,607	5	618,485	17
20 4 0,4	694,557	707,487	11	708,480	8	729,108	13
20 4 0,6	788,594	810,854	3	792,568	8	792,456	14
20 4 0,8	870,076	885,092	5	872,456	6	893,010	16
20 4 1,0	934,083	942,461	5	935,758	5	945,162	15

Table 2. ABC Result2

n p $\alpha$	optimum	Strategy 1		Strategy 2		Strategy 3	
		mean	score	mean	score	mean	score
25 2 0,2	996,022	996,022	30	996,022	30	996,022	30
25 2 0,4	1.072,489	1.072,489	30	1.072,489	30	1.072,489	30
25 2 0,6	1.137,081	1.137,081	30	1.137,081	30	1.137,081	30
25 2 0,8	1.180,020	1.180,020	30	1.180,020	30	1.180,020	30
25 2 1,0	1.206,620	1207,3408	28	1.206,620	30	1.206,620	30
25 3 0,2	752,907	764,524124	6	760,109	16	827,724	29
25 3 0,4	859,636	866,4589	10	863,085	17	859,636	30
25 3 0,6	949,230	951,231	24	951,293	17	988,116	28
25 3 0,8	1.020,037	1027,044	5	1021,154	20	1.020,037	30
25 3 1,0	1.062,144	1064,228	15	1063,694	19	1.062,144	30
25 4 0,2	618,483	651,358	6	642,984	5	641,820	13
25 4 0,4	754,489	772,788	11	757,599	5	774,604	12
25 4 0,6	866,445	895,458	9	869,789	5	872,457	10
25 4 0,8	951,755	965,150	8	962,3739	6	973,852	11
25 4 1,0	1.006,657	1008,987	10	1017,522	5	1010,985	12

### III. RESULTS

This algorithm was coded in MATLAB for all strategies and CAB (Civil Aeronautics Board) data set was used. This data set based on airline passenger flow between cities of United States. There are 25 nodes and collection cost  $\lambda$  and distribution cost  $\gamma$  are equal to one, while transferring cost  $\alpha$  takes from 0.2 to 1.0 [11].

The ABC algorithm was run 30 times on each instance. Different results were obtained in 20 and 25 cities and 2, 3, 4 hubs problem. These results are shown in Table 1 and Table 2. In these tables, optimum results obtained by mathematical model of each problem are given. The number of optimal solutions for Strategy 3 is higher than other strategies for all problems.

### IV. DISCUSSION

In this paper, UMAPHMP problem is solved via ABC algorithm with different search strategies. The performances of the strategies are tested using the CAB dataset. All strategies found optimal results. But it is seen that strategy 3 has better performance than other strategies. For each strategy, mean fitness values and the number of optimum found in 30 runs are given Table 1 and Table 2.

### V. CONCLUSION

Our study shows that the solution of the uncapacitated multiple allocation p-hub median problem can be done effectively by artificial bee colony algorithm. And the effectiveness of different search strategies for the algorithm are analyzed.

In future work, larger problems can be considered. Also ABC algorithm can be used to solve different types of hub location problems. And the performance of the ABC algorithm can be tested.

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